Name: _____ Application Number: _____

Instructions. You will have 45 minutes to answer the questions. This paper has fifteen questions, all carry equal weight.

1. How many times will the following pseudo-code print `hello` upon executing `fibonacci(5)`?

```
def fibonacci(n):

    if n <= 0:
        return 0
    elif n == 1:
        return 1

    print ("hello")

    return fibonacci(n - 1) + fibonacci(n - 2)
```

Write your answer here: _____

2. What is the mathematical function computed by the following function?

```
float eval (int x) { /* assume x >= 0 */
    int i;
    float temp = 4.2;
    float value = 1;
    if (x == 0) return 1;
    else
        for (i = 0 ; i < x; i++)
            value *= temp;
    return value;
}
```

Write your answer here: _____

3. What is the number of times the printf statement is executed in the following program?

```
for (i = 1 ; i < 5 ;  i++)
    for (j=i+1; j<6; j++)
       printf ("%d,%d", i, j) ;
```

Write your answer here: _____

4. Plot $f(x) = \frac{\sin(\pi x)}{x}$ as a function of x. Mark the maximum value, the place where this value is taken,

and a representative set of x values (on either side of the origin) where $f(x) = 0$.

5. The maximum of xe^{-x} is reached at $x^* =$ _____.

6. The integral $\int_{-1}^{1} \frac{1+x}{1+x^2} dx$ is equal to _____.

7. The set of points where $f(x) = |(x-1)^2(x-2)|$ is differentiable is _____.

8. The sum of the squares of the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 4 \\ 4 & 5 & 1 \end{bmatrix}$$

is _____.

9. The eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

are (write your answer in the space given below):

10. Which of the following choices hold true for the vectors $\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \end{bmatrix} \right\}$?
(You must tick all that apply.)

- ◯ Linearly independent
- ◯ Linearly dependent
- ◯ Neither linearly independent nor linearly dependent
- ◯ Orthogonal

11. Suppose x is an $n \times 1$ vector. The rank of the matrix xx^T is _____.

12. Let a biased coin be tossed n times in succession, with the probability of heads being p. The probability that all the tosses show the same face is _____.

13. Let X and Y be two correlated random variables with means μ_X and μ_Y, respectively. The mean of the random variable $X+Y$ is equal to _____.

14. Let X be an exponentially distributed random variable with mean $\lambda > 0$. If $x_1 > 0$ and $P(X > x_1) = \alpha$, then the conditional probability $P(X > 2x_1 | X > x_1)$ is _____.

15. Let X and Y be two discrete random variables with joint pmf:

$$P(X = x, Y = y) = \begin{cases} k(x+y), & x, y \in \{0,1,2\} \\ 0, & \text{otherwise} \end{cases}.$$

The probability $P(X = 0 | Y = 0) =$ _____.